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A UNIVERSAL TRANSITION FROM QUASI-PERIODICITY TO CHAOS

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Abstract

A common route to chaos in dissipative systems proceeds from periodic to quasi-periodic flow (with two independent frequencies). Then, in the absence of rotational symmetry, the system generally mode locks before becoming turbulent. Beyond these qualitative features, the numerous experiments that have examined this regime differ in detail. Dynamical system theory had made the occurrence of the above transitions plausible but has provided no nontrivial quantitative and model independent information.

This situation, on the theoretical side, has recently changed with a proposal on how to modify the experiments so as to make the transition to chaos occur in a quantitatively universal manner [1, 2]. The essence of our proposal follows from K.A.M. theory which is the weak coupling limit of the strong coupling problem relevant to the turbulent transition. In addition to the Rayleigh number, the experimenter must control a second parameter so as to maintain the frequency ratio in the quasi-periodic state at a fixed irrational value. The golden mean, $(\sqrt{5}-1)/2$, is the optimal ratio experimentally.

The universality, which is restricted to the low frequencies in the time series, is obtained under the above circumstances because the transition to chaos is continuous. In particular the singular low frequency structure in the spectrum develops continuously as $R \to R_T$ from below or as the frequency ratio approached $(\sqrt{5}-1)/2$ at R_T. These assertions are rigorously established by a renormalization group analysis that resembles the one developed by Feigenbaum to account for the universal features of period doubling.

We again stress that all the low frequency complex amplitudes obtained from either a fluid experiment or a forced nonlinear oscillator at the quasi-periodic to turbulent transition are universal. At present, the theoretical predictions are most easily derived numerically by iterating the map

$$\phi' = \phi + \omega_0 - \frac{a}{2\pi}\sin(2\pi\phi)$$

for $a = 1$ (corresponding to $R = R_T$) and adjusting ω_0 to achieve the desired rotation number.

References

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[2] S. Shenker, Physica 5D (1982) 405; M. Feigenbaum, L. Kadanoff and S.J. Shenker, preprint.